# LESSON 3.2a

## Introduction to Complex Numbers

**What do you think?**

Can you take the square root of a negative number?

- No … not with the numbers and number system we have been working with (***real*** numbers)

- …we can’t find a number that times itself results in a negative (only a positive times a negative gives a negative)

The thing is … trying to take the square root of a negative happens in many real-life situations:

- …in electrical circuits

- …in financial forecasting

- …in quantum physics

**Some fun math history**

Once upon a time (50AD) … there was a guy named Heron of Alexandria.

He was trying to find the volume of a section of a pyramid.

But…he ran into a problem: he got to a point in his calculations where he had $\sqrt{81 - 114}$

…and he stopped because like any good math student he knew you can't take the square root of a negative.

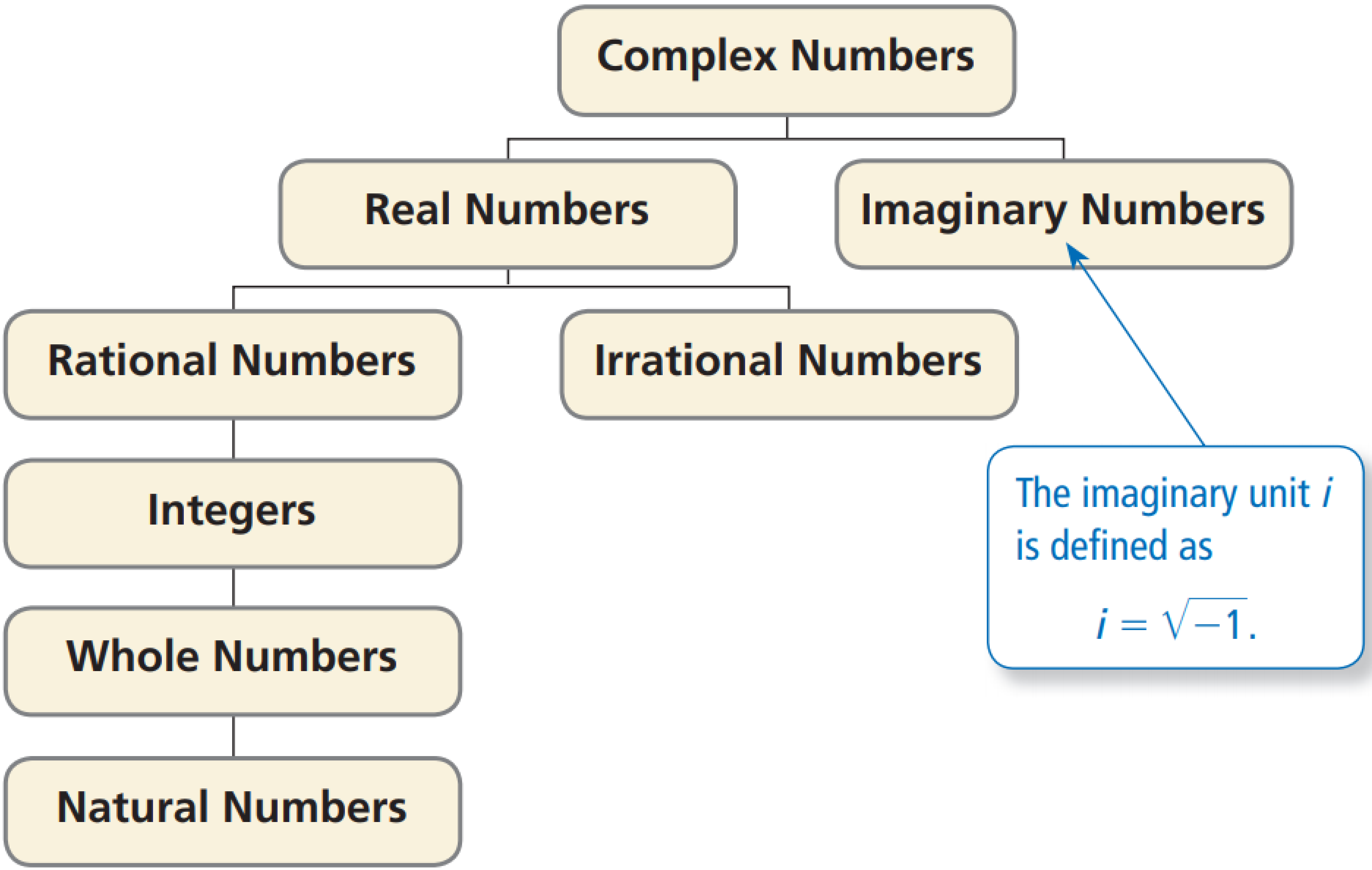
So he gave up and called this the "impossible pyramid."

However for the next 1500 years mathematicians and engineers kept bumping into situations where they had to take the square root of a negative.

They were getting frustrated and called these numbers (square roots of a negative) ***Imaginary Numbers***.

In 1545 guy named Girolamo Cardano (and later in 1637 Rene Descartes) figured out a way to make all this work in math…they came up with two new number systems:

1) the ***Imaginary Number System*** and 2) the ***Complex Number System***


Complex Numbers
Real Numbers
Imaginary Numbers
Rational Numbers
Irrational Numbers
Integers
Whole Numbers
Natural Numbers
The imaginary unit $i$ is defined as
$i = \sqrt{-1}$.

**Today you will:**

- Define and use the imaginary unit i.
- Add, and subtract, complex numbers

**Core Vocabulary:**

- imaginary unit i, p. 104
- complex number, p. 104
- imaginary number, p. 104
- pure imaginary number, p. 104

**The Imaginary Unit *i*:**

$i = \sqrt{-1}$

$i^2 = -1$

The imaginary unit *i* can be used to write the square root of any negative number.

| **The Square Root of a Negative Number Property**: | **Example** |
|---|---|
| 1. If $r$ is a positive real number, then $\sqrt{-r} = i\sqrt{r}$. | $\sqrt{-3} = i\sqrt{3}$ |
| 2. By the first property, it follows that $(i\sqrt{r})^2 = -r$ | $(i\sqrt{3})^2 = i^2 \cdot 3 = -3$ |

Find the square root of each number.

**a.** $\sqrt{-25}$ **b.** $\sqrt{-72}$ **c.** $-5\sqrt{-9}$

SOLUTION

**a.** $\sqrt{-25}$

A **complex number** written in *standard form* is a number $a + bi$ where $a$ and $b$ are real numbers.

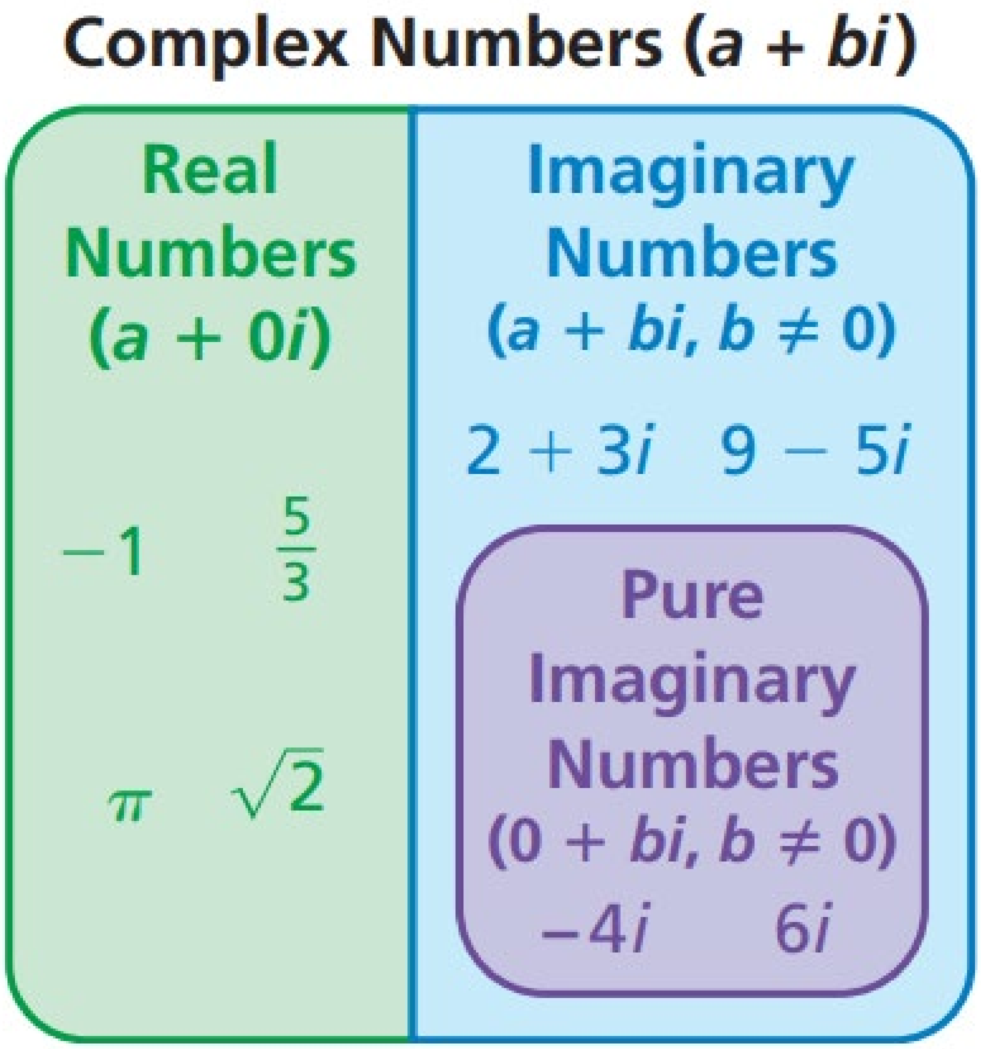
real part — imaginary part

If $b \neq 0$, then a $+ bi$ is an ***imaginary number***.

example: $3 + 2i$

If a $= 0$ and $b \neq 0$, then $a + bi$ is a ***pure imaginary number***.

example: $-6i$



**Equality of Complex Numbers**

Two complex numbers $a + bi$ and $c + di$ are equal if and only if $a = c$ and $b = d$

In other words, the correspond parts must be equal:

- the real parts
- and the imaginary parts

Find the values of $x$ and $y$ that satisfy the equation $2x - 7i = 10 + yi$.

SOLUTION

Set the real parts equal to each other and the imaginary parts equal to each other.

| | | | |
|---|---|---|---|
| $2x = 10$ | Equate the real parts. | $-7i = yi$ | Equate the imaginary parts. |
| $x = 5$ | Solve for $x$. | $-7 = y$ | Solve for $y$. |

So, $x = 5$ and $y = -7$.

**Start working on #13-20 on page 108 … you have 5 minutes.**

**Sums and differences of Complex Numbers**

…just add (or subtract) the corresponding parts (real and imaginary parts) separately.

**Sum of complex numbers:** $(a+bi)+(c+di)=(a+c)+(b+d)i$

**Difference of complex numbers:** $(a+bi)-(c+di)=(a-c)+(b-d)i$

Add or subtract. Write the answer in standard form.

**a.** $(8 - i) - (5 + 4i)$

**b.** $(7 - 6i) - (3 - 6i)$

**c.** $13 - (2 + 7i) + 5i$

**Start working on #21-30 on page 108 … you have 5 minutes.**

SOLUTION

**a.** $(8 - i) + (5 + 4i)$

$= 13 + 3i$ Write in standard form.

**b.** $(7 - 6i) - (3 - 6i)$

$= 4 + 0i$ Simplify.

$= 4$ Write in standard form.

**c.** $13 - (2 + 7i) + 5i$

$= (11 - 7i) + 5i$ Simplify.

$= 11 + (-7 + 5)i$ Definition of complex addition

$= 11 - 2i$ Write in standard form.

# Homework

Pg 108, #1-32